Math 155, ***Lecture Notes- Bonds*** **Name____________**

Section 9.3 *The Integral Test and p-Series*

In this section we will study series with positive terms. In particular, in this section we consider the connection between improper integrals, bounded areas over infinite intervals, and plots of partial sums sequences.

> **THEOREM 9.10 The Integral Test**
>
> If f is positive, continuous, and decreasing for $x \geq 1$ and $a_n = f(n)$, then
>
> $$\sum_{n=1}^{\infty} a_n \quad \text{and} \quad \int_1^{\infty} f(x)\,dx$$
>
> either both converge or both diverge.

Ex. 1: Apply the Integral Test: $\sum_{n=1}^{\infty} \frac{1}{n^2}$

Ex. 2: Apply the Integral Test: $\sum_{n=1}^{\infty} \frac{1}{n}$

p-Series and Harmonic Series

There are two types of series that we should be able to recognize by inspection and we should be able to determine the convergence, or divergence of these series very quickly. We need to be able to do this in order to use this information for "comparison" purposes. In the next section we begin to understand series behavior by making use of comparing particular series to the known behavior of "simpler" series. But first, we need a few definitions.

p-series: $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots$

the harmonic series: $\sum_{n=1}^{\infty} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$

the general harmonic series: $\sum_{n=1}^{\infty} \frac{1}{an+b}$

THEOREM 8.5 A Special Type of Improper Integral

$$\int_1^{\infty} \frac{dx}{x^p} = \begin{cases} \frac{1}{p-1}, & \text{if } p > 1 \\ \text{diverges}, & \text{if } p \le 1 \end{cases}$$

THEOREM 9.11 Convergence of *p*-Series

The p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdot \cdot \cdot$$

1. converges if $p > 1$, and

2. diverges if $0 < p \le 1$.

Ex. 3: Determine convergence: $\sum_{n=1}^{\infty} \frac{2}{\sqrt[5]{n^2}}$

Ex. 4: Determine convergence: $1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\frac{1}{5\sqrt{5}}+\cdots$

Ex. 5: Determine convergence: $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

Ex. 6: Determine convergence: $\sum_{n=1}^{\infty} n e^{-\frac{n}{2}}$